On using automated deduction techniques in dynamic geometry environments

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What is Dynamic Geometry (DG)?

Models built by computer software that can be changed dynamically

King and Schattschneider (1997)

Properties of dynamical models

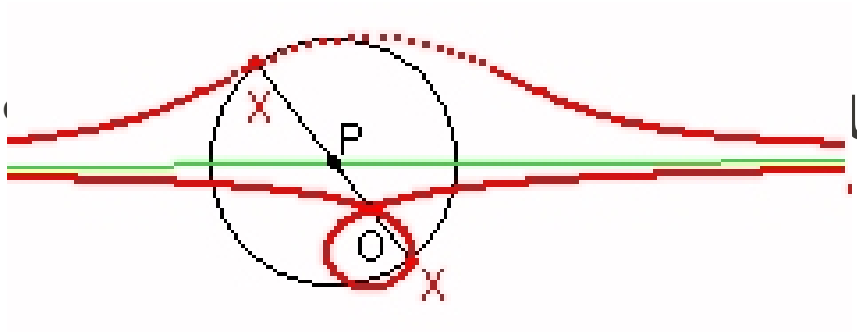
- Dynamic transformations
- Dynamic measurement
- Free dragging
- Animation
- Locus generation

Common strategy for tracing loci

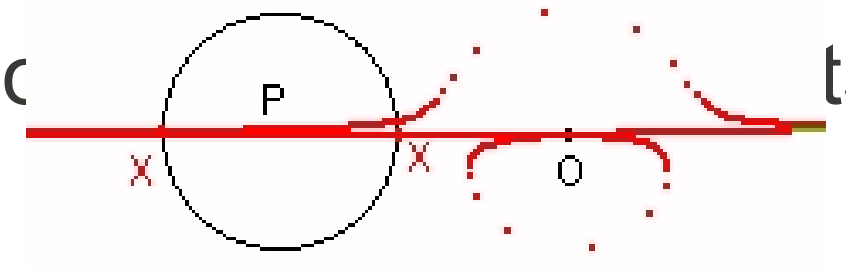
In order to trace the locus of an object X, depending somehow on another object P:

- Select the object P, the *driver object*, which must lie on a predefined path

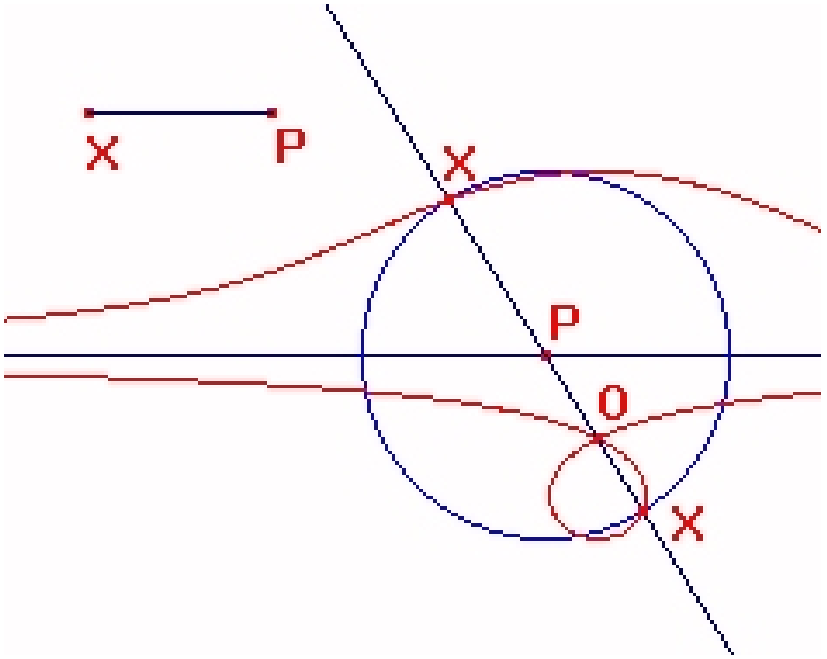
- Sample the path and plot X for each sample

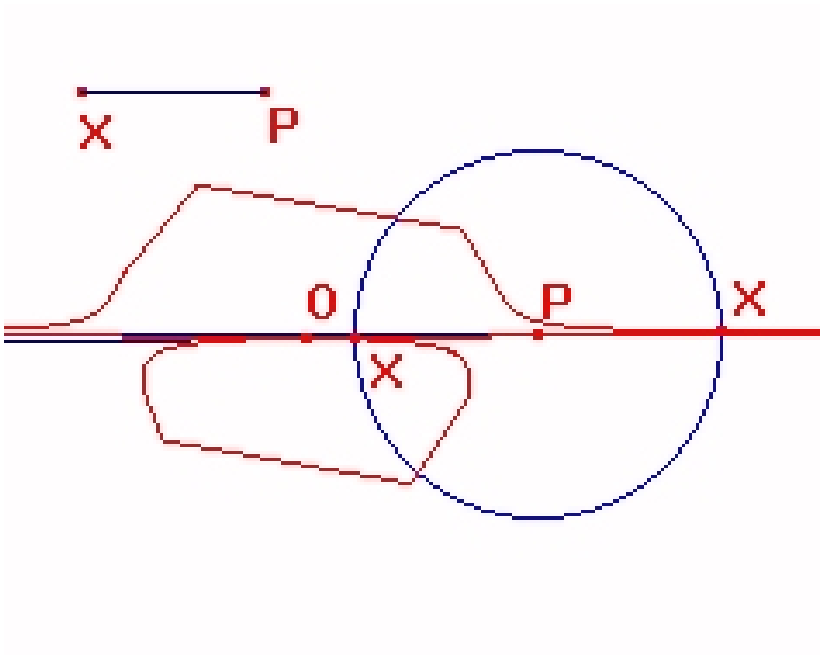


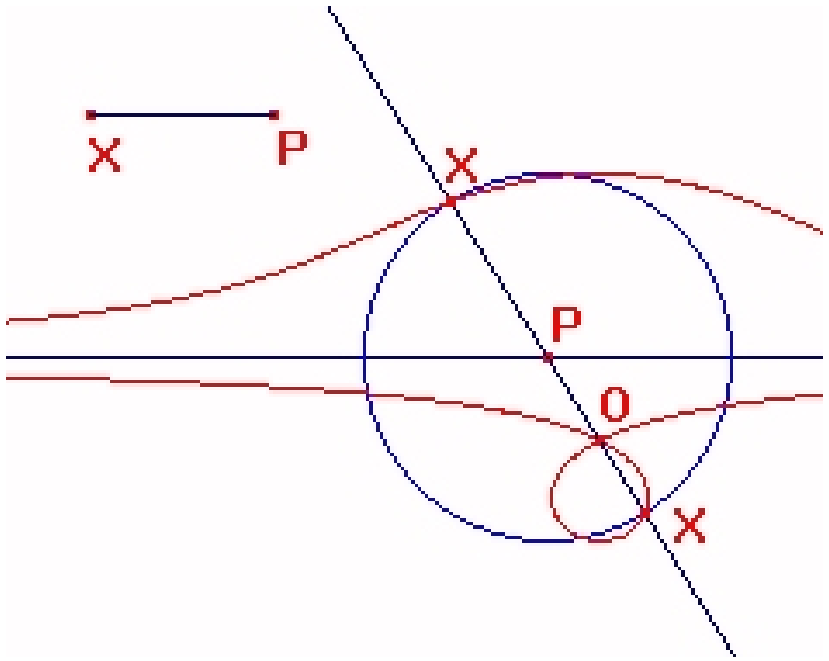
uc



ts


X
P
X
P
O
X


X
P
O
P
X
X



$P(x_1,x_2)$ lies on the x axis:

$$x_2=0$$

O(0,-1) is a fixed point

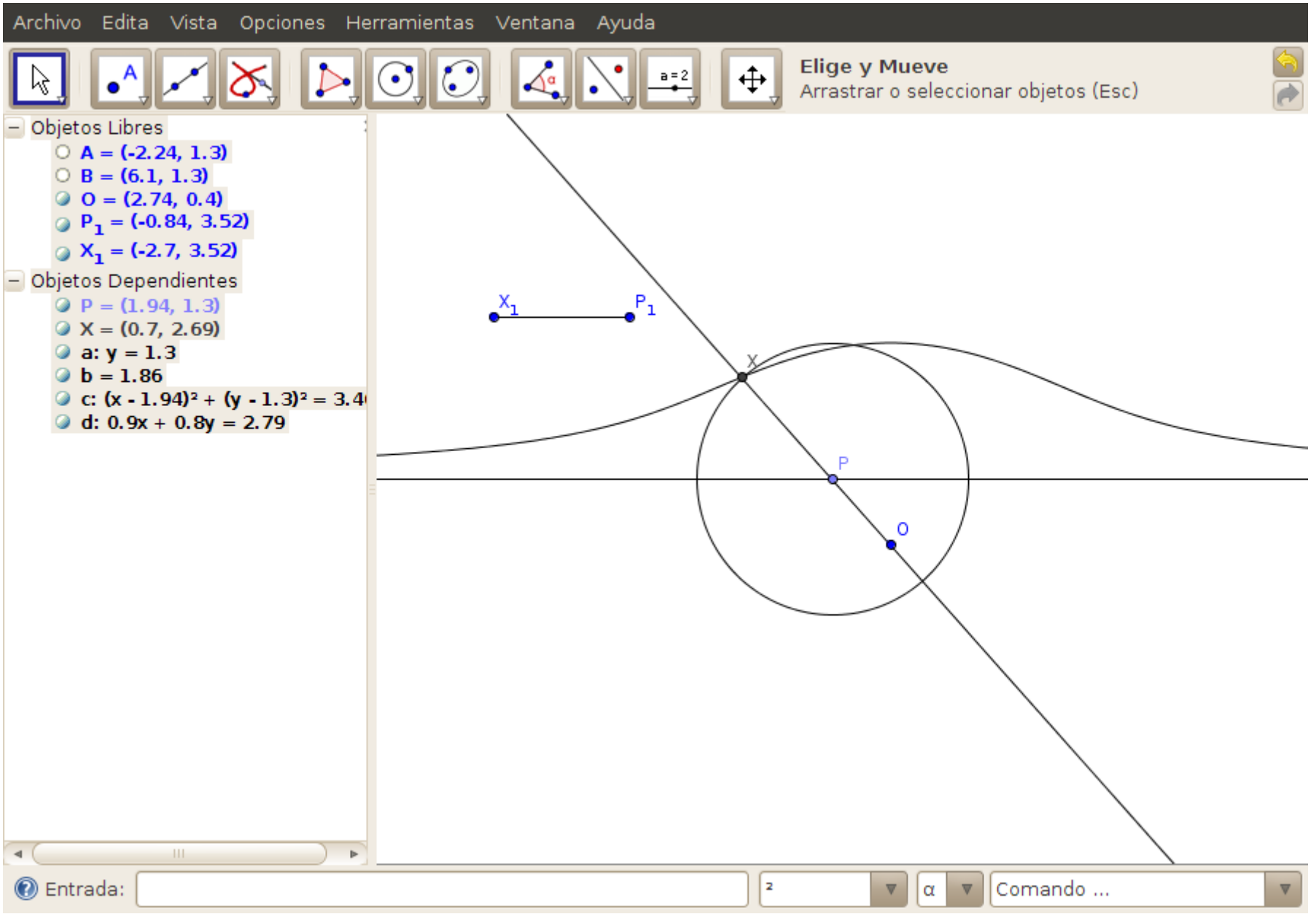
X(u,v) lies on the line OP:

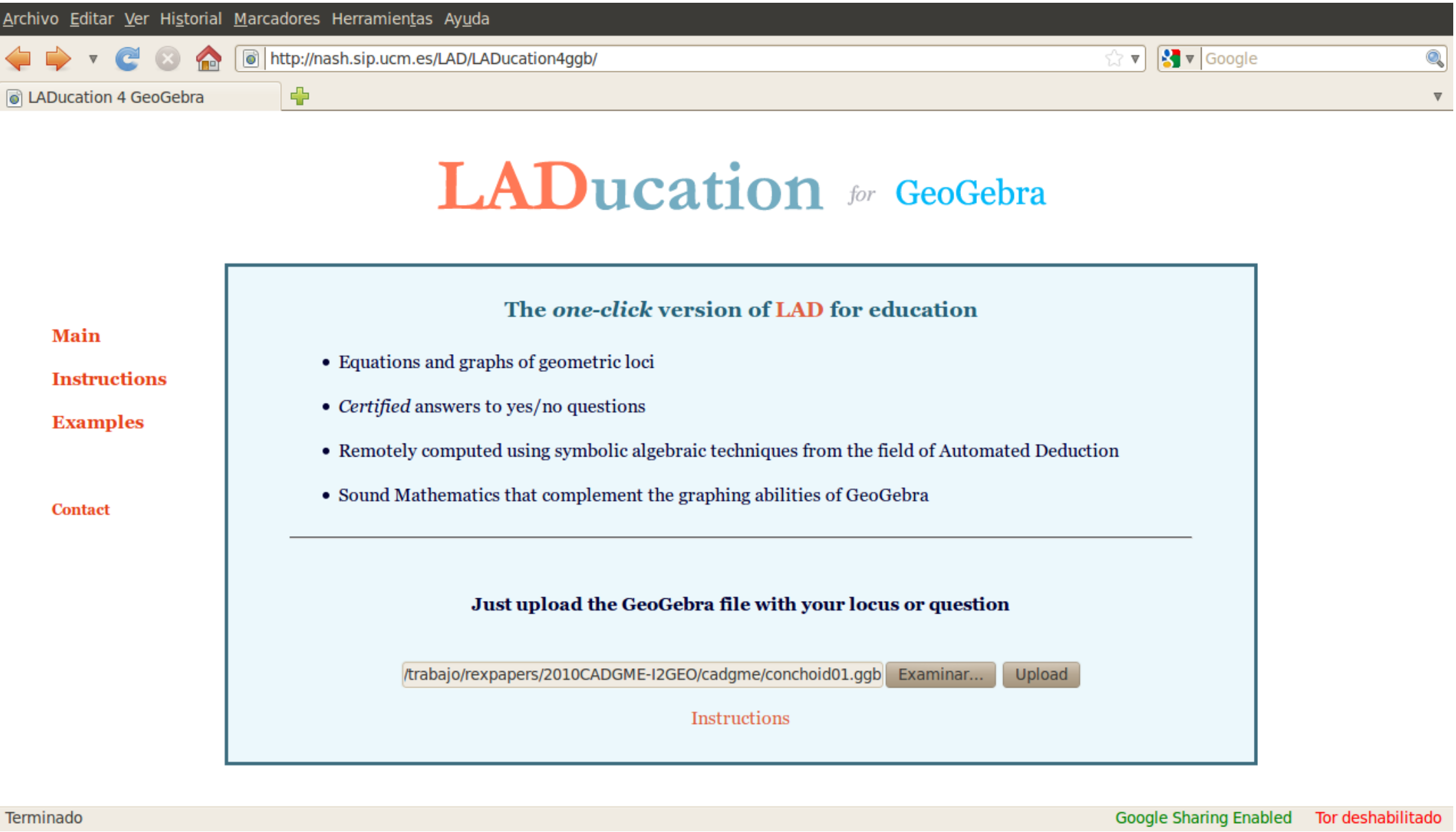
$$(v+1)/u=1/x_1$$

and on the circle (O,d):

$$(u-x_1)^2+v^2=d^2$$

Eliminating x_1 in the above equations we get the conchoid's equation!


Archivo Edita Vista Opciones Herramientas Ventana Ayuda
Elige y Mueve
Arrastrar o seleccionar objetos (Esc)
Objetos Libres
A = (-2.24, 1.3)
B = (6.1, 1.3)
O = (2.74, 0.4)
P1 = (-0.84, 3.52)
X1 = (-2.7, 3.52)
Objetos Dependientes
P = (1.94, 1.3)
X = (0.7, 2.69)
a: y = 1.3
b = 1.86
c: (x - 1.94)² + (y - 1.3)² = 3.4
d: 0.9x + 0.8y = 2.79
X1
P1
X
P
O
Entrada:
Comando ...


Archivo Editar Ver Historial Marcadores Herramientas Ayuda
http://nash.sip.ucm.es/LAD/LADucation4ggb/
Google
LADucation 4 GeoGebra
LADucation for GeoGebra
Main
Instructions
Examples
Contact
The one-click version of LAD for education
• Equations and graphs of geometric loci
• Certified answers to yes/no questions
• Remotely computed using symbolic algebraic techniques from the field of Automated Deduction
• Sound Mathematics that complement the graphing abilities of GeoGebra
Just upload the GeoGebra file with your locus or question
/trabajo/rexpapers/2010CADGME-I2GEO/cadgme/conchoid01.ggb
Examinar...
Upload
Instructions
Terminado
Google Sharing Enabled
Tor deshabilitado

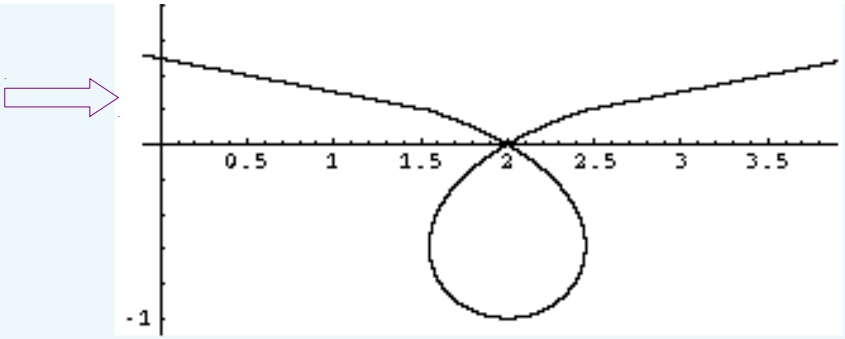
GeoGebra Help

Official Manual 3.2



Continuity

GeoGebra allows you to turn the continuity heuristic *On* or *Off* in the *Options* menu. The software uses a near-to-heuristic to keep moving intersection points (e. g., line-conic, conic-conic) close to their old positions and avoid jumping intersection points.



The locus equation is

```
the quartic 4 - 4*x + x^2 - 8*y + 8*x*y - 2*x^2*y + y^2 - 4*x*y^2 + x^2*y^2 - 2*y^3 +
  y^4 == 0
```

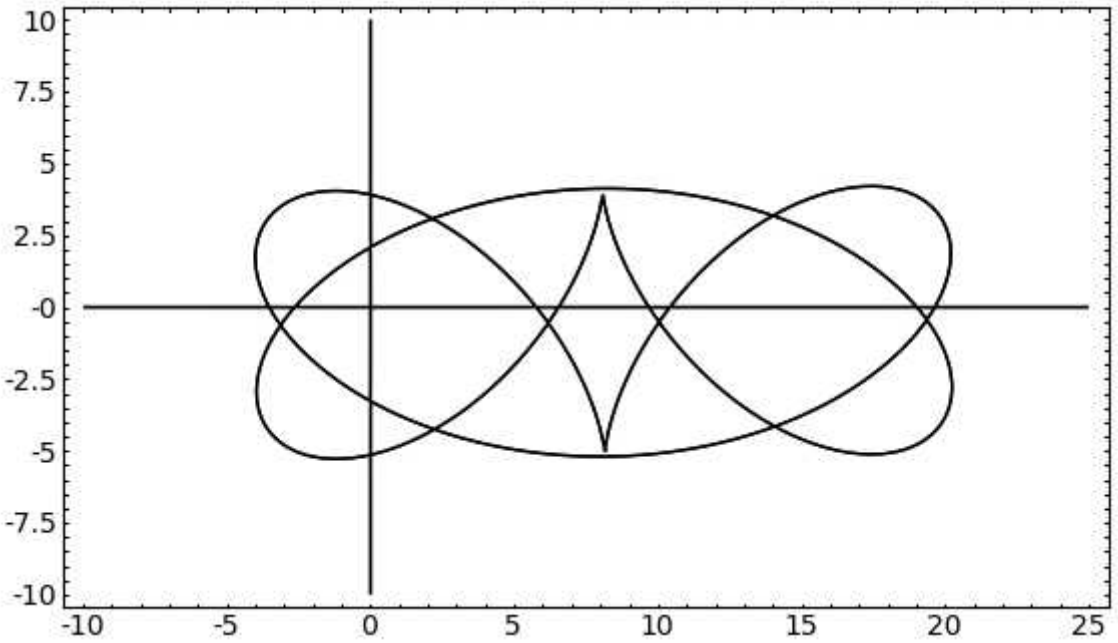
GeoGebra & Google Summer of Code 2010

Locus line equation

Find the equation for a locus line constructed in GeoGebra. The goal is an integrated solution that doesn't require upload of a ggb file to a server.

Contour and Implicit Plotting

Implement contour and implicit plotting routines in GeoGebra.

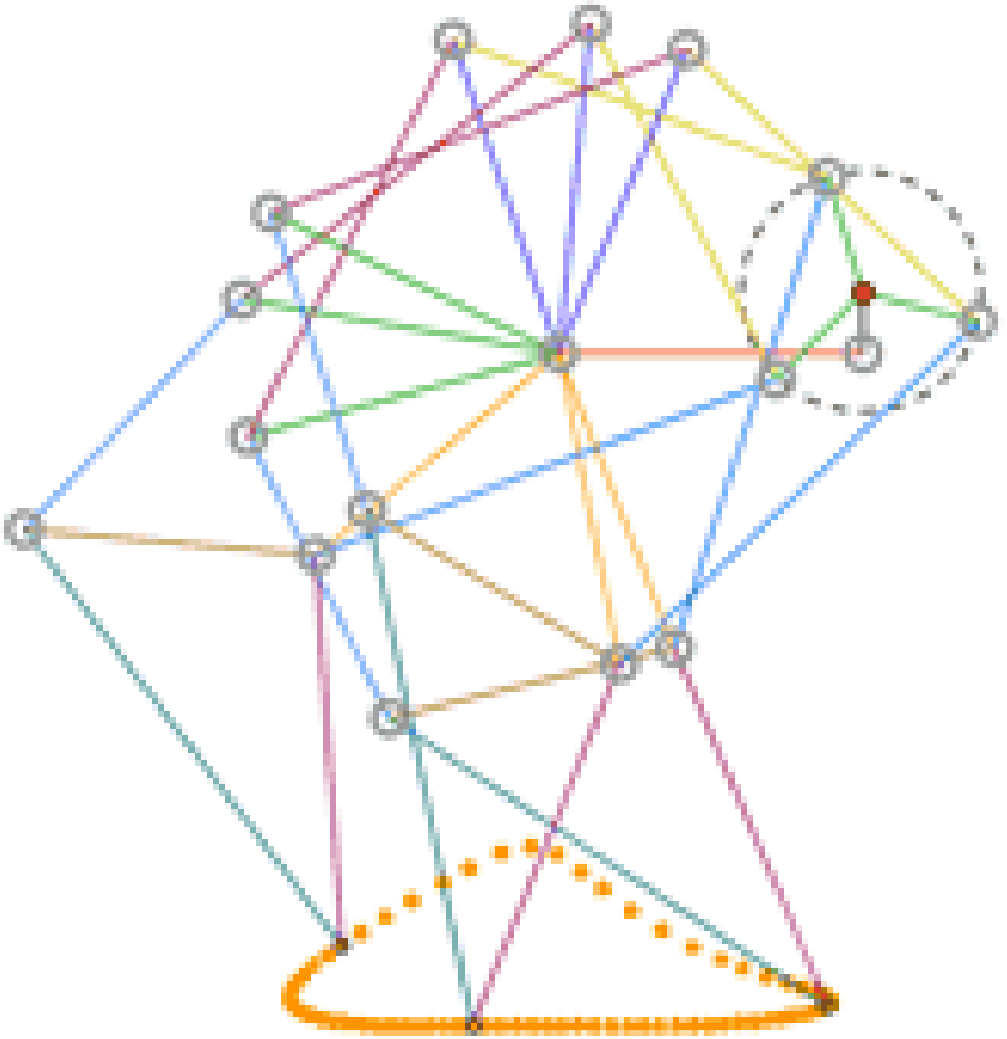

10
7.5
5
2.5
-0
-2.5
-5
-7.5
-10
-10
-5
0
5
10
15
20
25

$$
\begin{aligned}
&802329643516954820900526685891411759461151603 21 + \\
&27070348580320295122778438727197196015346056 00*x - \\
&12094748935307324983015398495218909628691240000*x^2 + \\
&13619198984264936387593375053095271088560000 00*x^3 + \\
&39415417758760375600583842450097208060000000 0*x^4 - \\
&10462622035598145245812073856421320000000000 0*x^5 + \\
&95819595447740757219532213549400000000000000*x^6 - \\
&39253765735440433237389630000000000000000000*x^7 + \\
&60457589458568094975062500000000000000000*x^8 - \\
&\text{(} \mathbf{30\ terms} \text{)} \\
&25919487370774725584999400000000000000000*y^7 + \\
&60457589458568094975062500000000000000000*y^8=0
\end{aligned}
$$

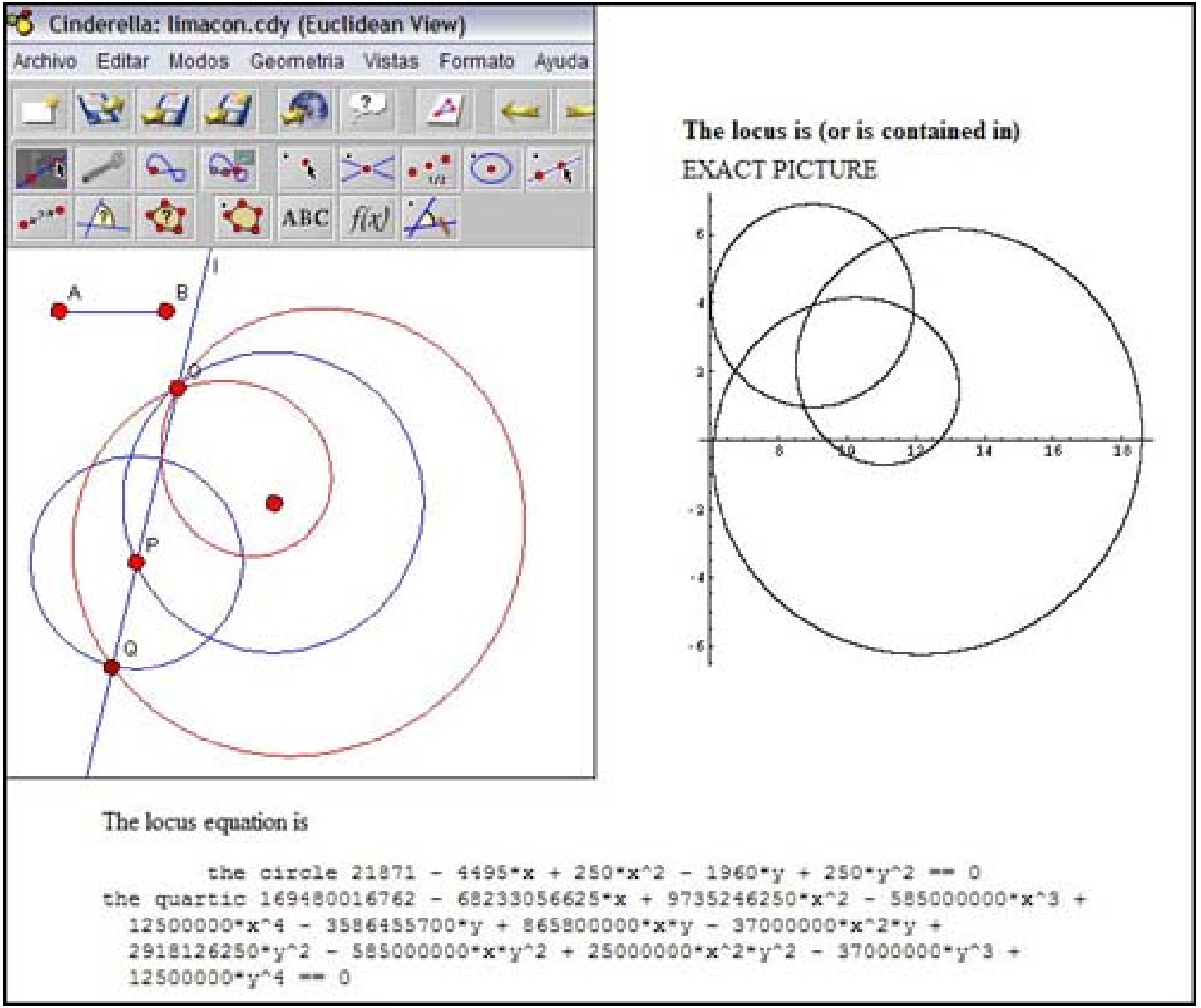
Should we hide this (kind of) result to students?

A case of failure





Loci and degenerated conditions



Extension to 3D



Range[-2.7,2.8,-4.4,2.8,-2.7,2.7];
FreePointD[Pt1,1,1,0];
FreePointD[Pt2,-31/49,16/9,0];
SphereD[Sp3,Pt1,Pt2];
PointOnSphere[Pt4,Sp3];
LineD[Ln5,Pt2,Pt4];
SphereD[Sp6,Pt4,Pt1];
Intersection1LineSphere[Pt8,Sp6,Ln5];
Intersection2LineSphere[Pt9,Sp6,Ln5];
Locus[O10,Pt8,Pt4];

OpenMath

- Extending the 3dgeo experimental CDs
- Requiring 3d-DGS developers OM-export

3D Automatic Discovery using CoCoA

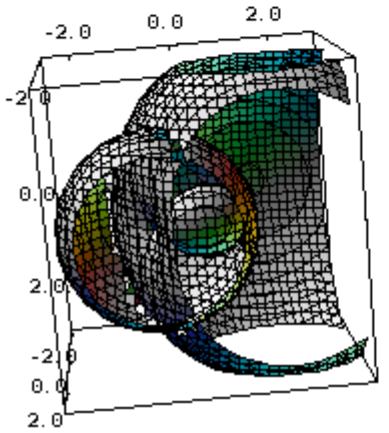


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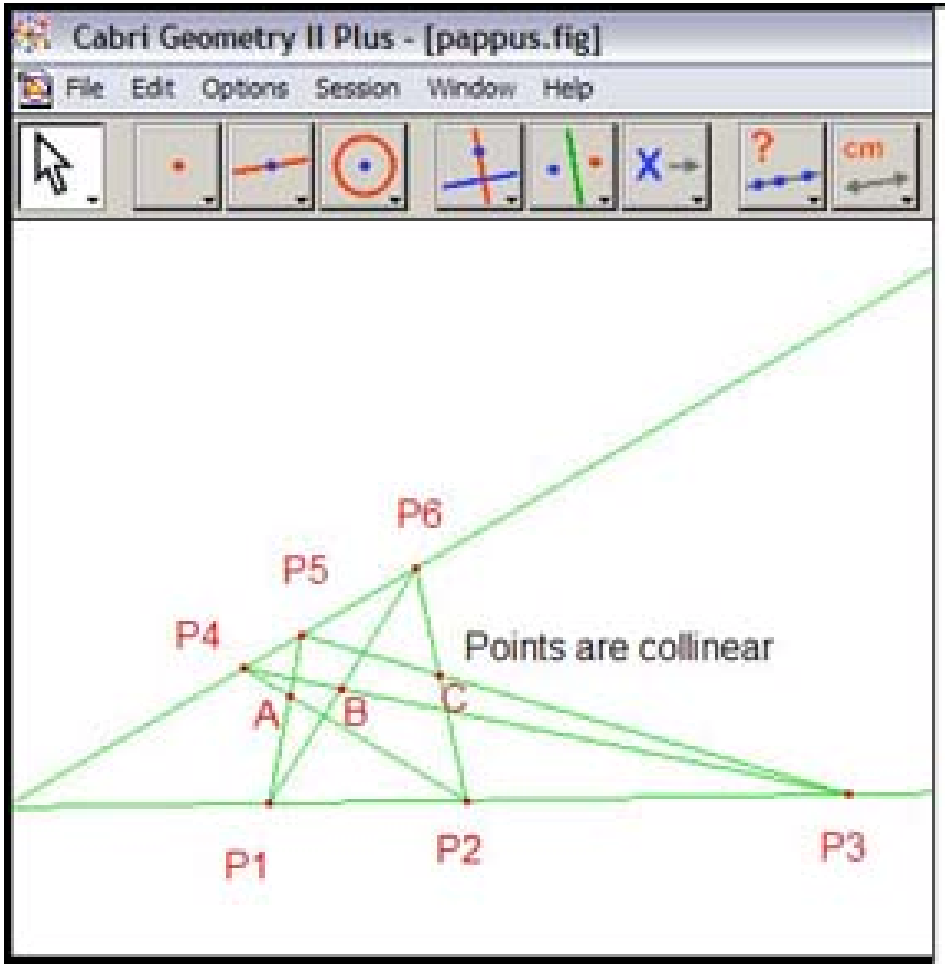
The locus of Pt8 in the given construction is

the sphere $441\,x^2+558\,x+441\,y^2+441\,z^2-1568\,y+128=0$

and $85766121\,x^4-343064484\,x^3+171532242\,y^2\,x^2+171532242\,z^2\,x^2-343064484\,y\,x^2-155363859\,x^2-343064484\,y^2\,x-343064484\,z^2\,x+686128968\,y\,x+80946126\,x+85766121\,y^4+85766121\,z^4-343064484\,y^3-155363859\,y^2+171532242\,y^2\,z^2-343064484\,y\,z^2-498428343\,z^2+1433186300\,y-860340224=0$



Proof

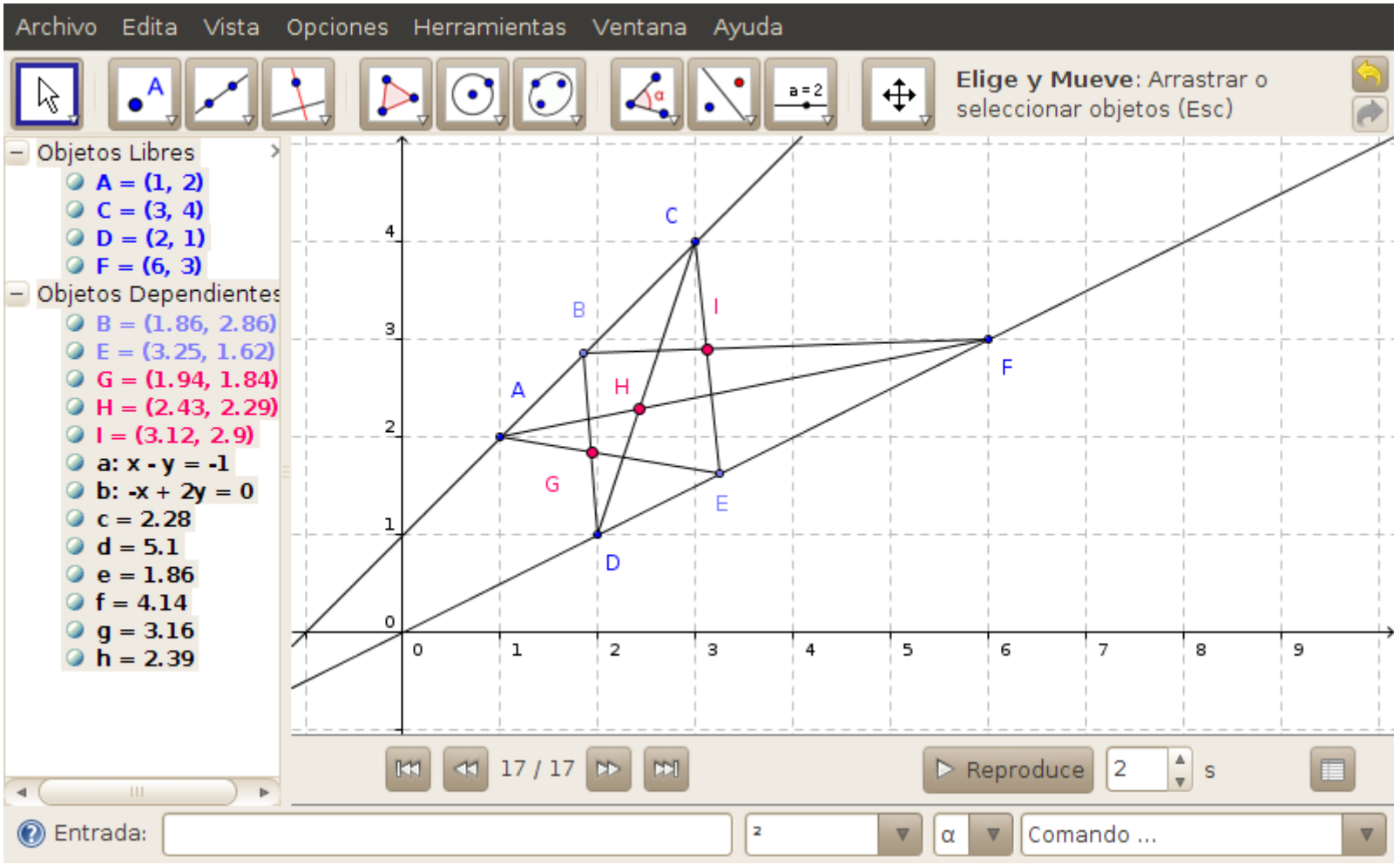



```
Given a construction with points
P4{-469/100,21/100}
P6{-274/100,134/100}
P1{-445/100,-111/100}
P3{210/100,-1}
P5{x[1],x[2]}
P2{x[3],x[4]}
A{x[5],x[6]}
B{x[7],x[8]}
C{x[9],x[10]}

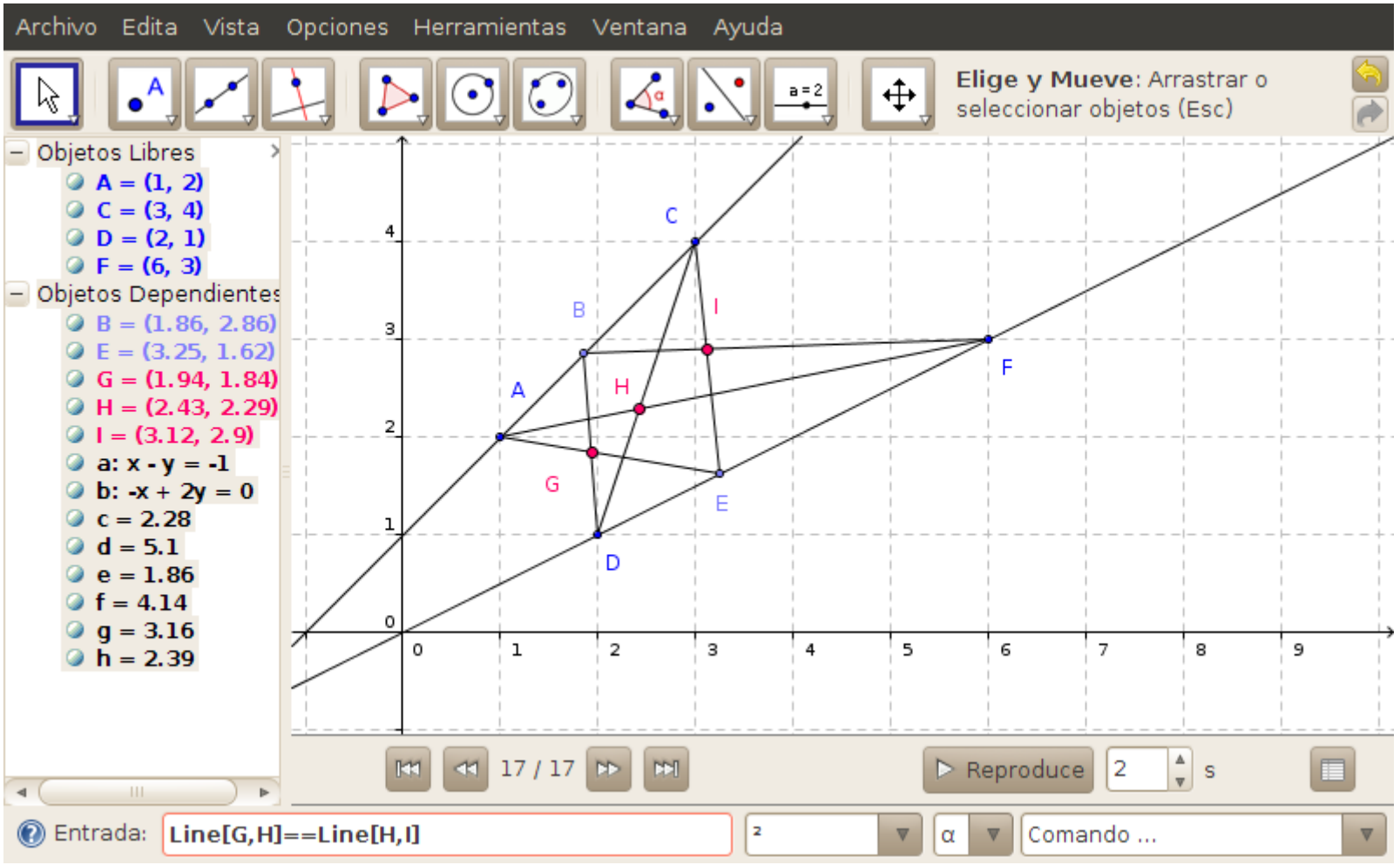
with constraints
Aligned(P5,P4,P6)
Aligned(P2,P1,P3)
Aligned(A,P5,P1)
Aligned(A,P2,P4)
Aligned(B,P4,P3)
Aligned(B,P6,P1)
Aligned(C,P5,P3)
Aligned(C,P6,P2)

the statement
 Aligned(A,B,C)
is true
```

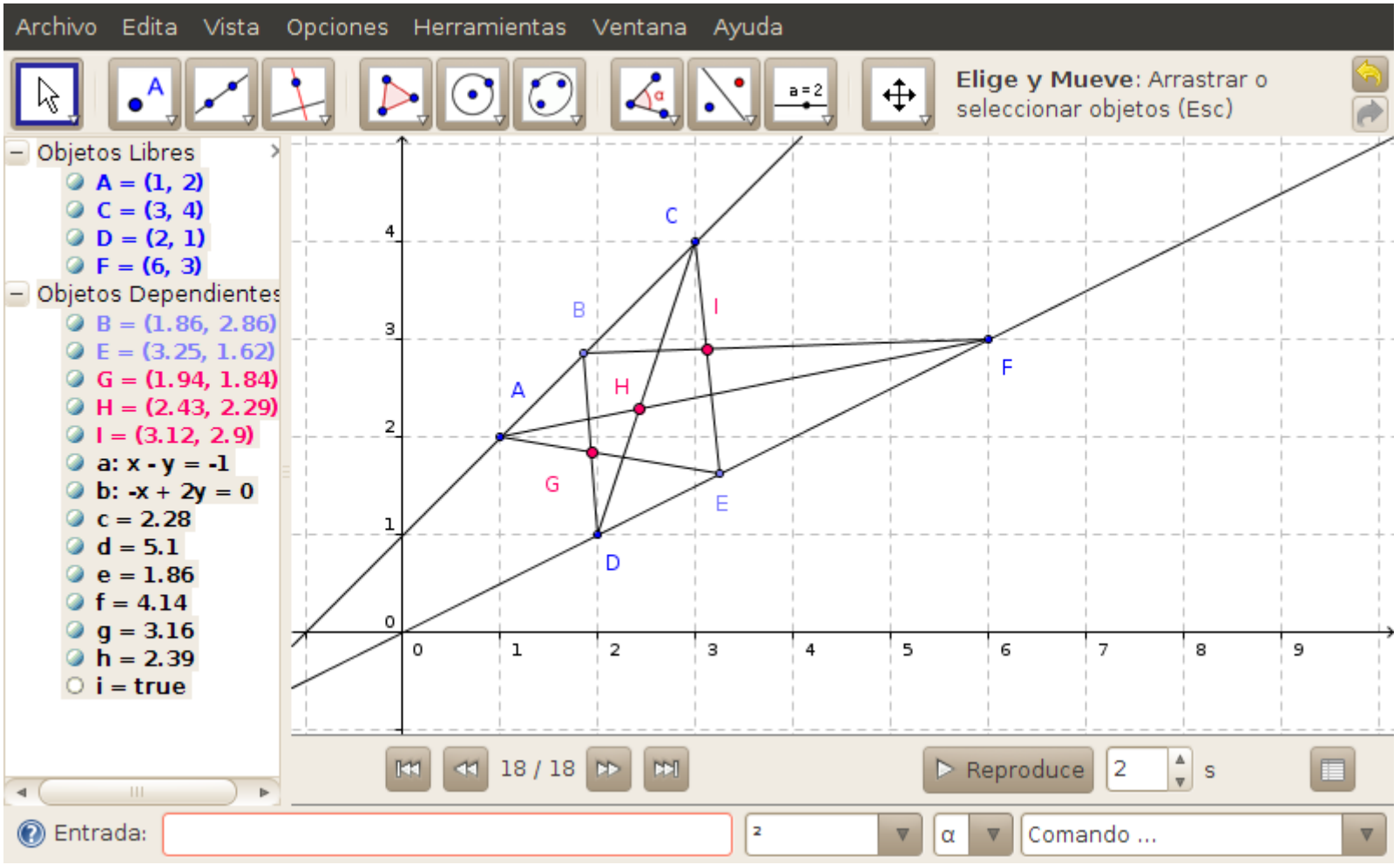
Proof


Archivo Edita Vista Opciones Herramientas Ventana Ayuda
Elige y Mueve: Arrastrar o seleccionar objetos (Esc)
Objetos Libres
A = (1, 2)
C = (3, 4)
D = (2, 1)
F = (6, 3)
Objetos Dependientes
B = (1.86, 2.86)
E = (3.25, 1.62)
G = (1.94, 1.84)
H = (2.43, 2.29)
I = (3.12, 2.9)
a: x - y = -1
b: -x + 2y = 0
c = 2.28
d = 5.1
e = 1.86
f = 4.14
g = 3.16
h = 2.39
A
B
C
D
E
F
G
H
I
17 / 17
Reproduce
2
s
Entrada:
Comando ...

Proof


Archivo Edita Vista Opciones Herramientas Ventana Ayuda
Elige y Mueve: Arrastrar o seleccionar objetos (Esc)
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g = 3.16
h = 2.39
A
B
C
D
E
F
G
H
I
17 / 17
Reproduce
2
s
Entrada:
Line[G,H]==Line[H,I]
Comando ...

Proof


Archivo Edita Vista Opciones Herramientas Ventana Ayuda
Elige y Mueve: Arrastrar o seleccionar objetos (Esc)
Objetos Libres
A = (1, 2)
C = (3, 4)
D = (2, 1)
F = (6, 3)
Objetos Dependientes
B = (1.86, 2.86)
E = (3.25, 1.62)
G = (1.94, 1.84)
H = (2.43, 2.29)
I = (3.12, 2.9)
a: x - y = -1
b: -x + 2y = 0
c = 2.28
d = 5.1
e = 1.86
f = 4.14
g = 3.16
h = 2.39
i = true
A
B
C
D
E
F
G
H
I
18 / 18
Reproduce
2
s
Entrada:
Comando ...

LADucation for GeoGebra

Main

Instructions

Examples

Contact

Your Geogebra file Pappus.ggb has been successfully uploaded.
It will be processed in 8 seconds.

```
Given a construction with points
A(1.,2.)
C(3.,4.)
D(2.,1.)
F(6.,3.)
B(x[1],x[2])
E(x[3],x[4])
H(x[5],x[6])
G(x[7],x[8])
I(x[9],x[10])

and with constraints
Aligned(B,A,C)
Aligned(E,D,F)
Aligned(H,A,F)
Aligned(H,C,D)
Aligned(G,A,E)
Aligned(G,B,D)
Aligned(I,B,F)
Aligned(I,C,E)

the statement
Aligned(G,H,I)
is true (except in certain degenerate cases)
```

Proof

(roughly) A proposition is declared to be True if

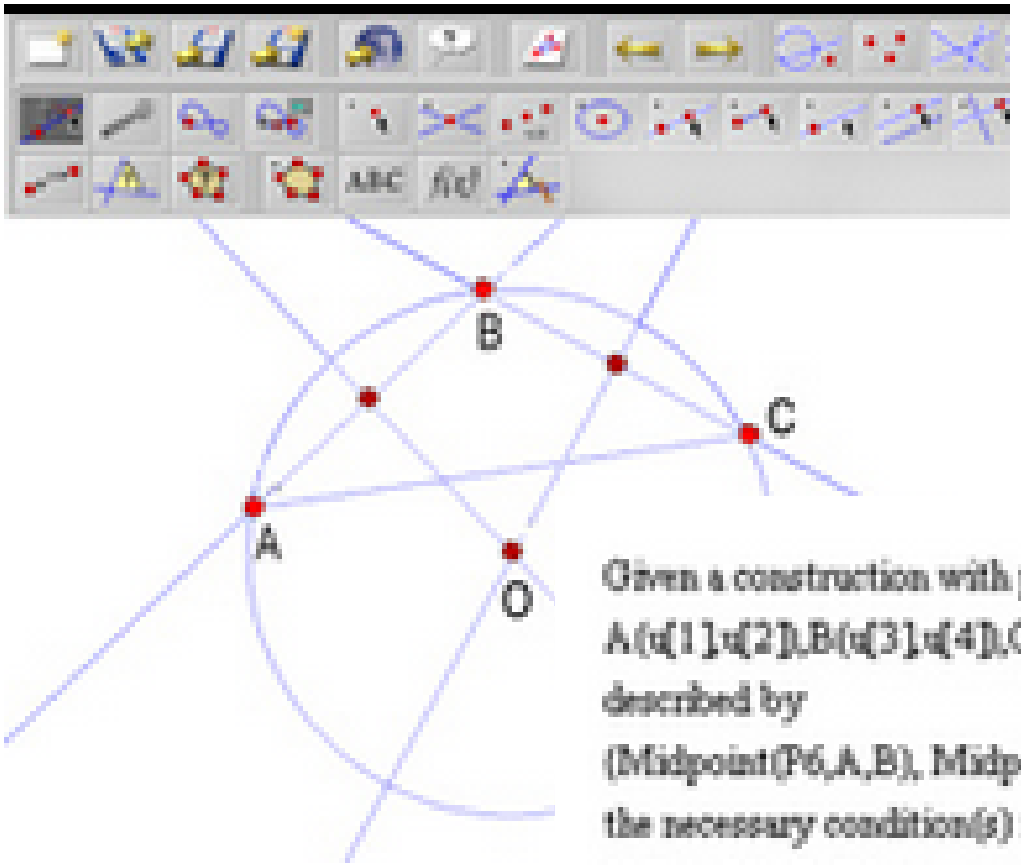
(any component of)

the hypotheses ideal is contained in the thesis ideal.

Be careful, this holds in the complex field, not **always** in the real one.

Sat(H,Sat(H,T)) <> (1)

Discovery

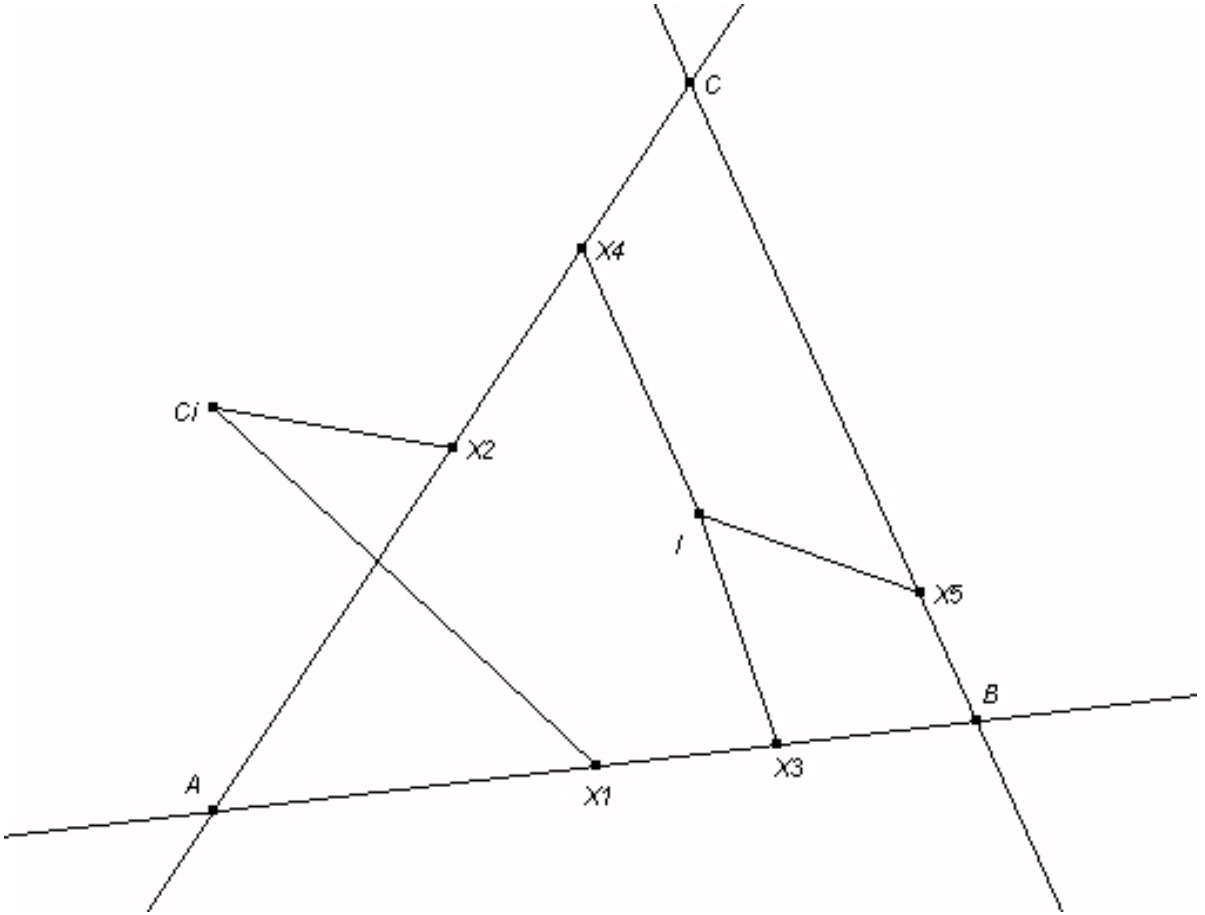


Given a construction with points
A(u[1],u[2]),B(u[3],u[4]),C(u[5],u[6]),P6(u[1],u[2]),P7(u[3],u[4]),O(u[5],u[6]),
described by
(Midpoint(P6,A,B), Midpoint(P7,B,C), Perpendicular(O,P6,A,B), Perpendicular(O,P7,B,C)),
the necessary condition(s) for
(Aligned(A,O,C))
is(are)

(Perpendicular(A, B, B, C)) V (Equal(A, C))

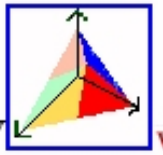
A formula from Euler

concerning the relation between the radii of the incircle, r, and the circumcircle, c, of a triangle, and the distance d between their centers.



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Given a construction with points
A(0,0),B(1,0),C(u[5],u[6]),X1(x[1],x[2]),X2(x[3],x[4]),X3(x[5],x[6]),X4(x[7],x[8]) ,X5(x[9],x[10]),I(u[7],u[8]),Ci(u[9],u[10]) ,
described by
{Midpoint(X1,A,B) , Midpoint(X2,A,C), Aligned(X3,A,B), Aligned(X4,A,C), Aligned(X5,B,C) },
the necessary condition(s) for
{Perpendicular(A,B,X1,Ci), Perpendicular(A,C,X2,Ci) , Perpendicular(A,B,I,X3),
Perpendicular(A,C,I,X4), Perpendicular(B,C,I,X5), distance(I,X3)=distance(I,X4), distance(I,X3)=distance(I,X5) }
is(are)

$$2u(9)-1==0 \wedge u(5)^2-u(5)+u(6)^2-2u(6)u(10)==0 \wedge (\{\text{Aligned}\{A,B,C\}\} \vee -u(7)^2+2u(5)u(7)+u(8)^2-u(5)+2u(6)u(8)-4u(8)u(10)==0) \wedge$$
$$(\{\text{Aligned}\{A,B,C\}\} \vee 2u(7)u(6)-u(6)-2u(5)u(8)-2u(7)u(8)+2u(8)==0) \wedge$$
$$(\{\text{Aligned}\{A,B,C\}\} \vee 2u(8)^3-8u(10)u(8)^2+2u(7)^2u(8)-2u(7)u(8)-2u(8)+u(6)==0) \wedge$$
$$(\{\text{Aligned}\{A,B,C\}\} \vee 2u(7)^3-3u(7)^2+2u(8)^2u(7)-8u(8)u(10)u(7)-u(8)^2+u(5)+4u(8)u(10)==0) \wedge$$
$$(\{\text{Aligned}\{A,B,C\}\} \vee -4u(8)^4+16u(10)u(8)^3-8u(6)u(10)u(8)^2+4u(8)^2-4u(6)u(8)+u(6)^2==0) \wedge$$
$$(\{\text{Aligned}\{A,B,C\}\} \vee -8u(7)u(8)^3+4u(8)^3-16u(5)u(10)u(8)^2+16u(7)u(10)u(8)^2-6u(5)u(8)+2u(7)u(8)+2u(8)+2u(5)u(6)-u(6)==0)$$

Try another Discovery?

d^4 - 2*d^2*c^2 - 4*r^2*c^2 + c^4

Dík
Gracias